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MAGNETOHYDRODYNAMIC SHOCK PROPAGATION IN NON-UNIFORM DUCTS

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MAGNETOHYDRODYNAMIC SHOCK PROPAGATION IN NON-UNIFORM DUCTS*

Roy M. Gundersen

1. Introduction

In recent years, there has been a great deal of interest in shock propagation in non-uniform ducts. Chester [1] determined the disturbance produced behind a shock of arbitrary strength propagating through a tube of arbitrarily varying cross-section by linearizing the problem on the basis of small area variations and found that the pressure perturbation behind the shock was given by

$$- K(P_2 - P_1)[\Delta A]/A \ ,$$

where $P_2 - P_1$ was the initial pressure discontinuity across the shock, $[\Delta A]$ the net change in area and the parameter K a monotonically decreasing function of the shock strength. Chester started with the full three-dimensional equations of motion and then carried out an averaging process by restricting the final consideration to the average pressure. However, such a restriction at the end means that the same results must be obtainable by starting with a one-dimensional analysis. This important fact was first realized by Paul Germain, and Chester's results obtained by a one-dimensional analysis by Gundersen [6], extracts from which appear in [7], the analysis being based on techniques presented by

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Germain and Gundersen [5]. Surprisingly, the one-dimensional approach leads to an improvement, namely, the term $[\Delta A]/A$ in Chester's work is replaced by $\bar{A}/A$ where $\bar{A}$ is the perturbed area distribution. This shows the relation between the area change and shock strength at any point. This fact was observed by Chisnell [2],who obtained it in a different manner, essentially assuming Chester's steady state solution,valid for large time only, could be utilized, and used to apply an integrated form of Chester's solution to discuss converging cylindrical and spherical shocks with great accuracy.

Lately, many others have worked on similar or related problems, e.g., Rosciszewewski [8] and Whitham [9] who also rederived Chester's results.

In the present paper, the techniques presented in [7] are generalized to determine the perturbation produced when an initially plane magnetohydrodynamic shock wave encounters an area variation, the problem being linearized on the basis of small area variations. In order to ensure that all changes in the behavior of the shock are caused by the non-uniform area, it is assumed that the area variations are confined to the region $x > 0$ while to the left of the section $x = 0$ (say), the tube is of constant area, and the initially plane shock propagates through this portion with constant speed. The fluid in front of the shock is assumed at rest.

When the shock meets the area variation, it is perturbed, the shock strength altered, and the subsequent flow is non-isentropic.

There are two distinct contributions to the disturbance, namely, a permanent perturbation caused directly by the area change and a transient disturbance, due to reflection from the shock of the permanent perturbation,

which propagates with velocity $\omega = [c^2 + b^2]^{1/2}$, where c is the local speed of sound and b the Alfven speed, with respect to the flow behind the shock.

In the immediate vicinity of the shock, the ultimate effect is an altered shock strength and concomitant pressure change behind the shock. When the flow behind the shock is $< \omega$, there is a disturbance convected to the left with velocity ω relative to the fluid while for flow $> \omega$, a similar disturbance is generated which propagates to the right with velocity ω relative to the fluid. This latter must be added to the steady flow solution. Expressions for these various contributions are obtained.

Specifically, it is found that the pressure perturbation immediately behind the incident shock is given by:

$$\overline{P}_2 = - K_2(\sigma, m_1)(P_2 - P_1)\overline{A}_2/A_2$$

where perturbations are denoted by a bar, $P_2 - P_1$ is the original pressure discontinuity across the shock, A_2 the cross-sectional area, σ the shock strength defined as a density ratio and $m_1 = b_1/c_1$ a measure of the applied field.

For all m_1, $\lim\limits_{\sigma \to 1+} K_2 = 0.5$ and $\lim\limits_{\sigma \to 4-} K_2 = 0.45085$ [the present analysis is limited to a monatomic gas so that $1 \leq \sigma \leq 4$]. But these are exactly the limits for the ordinary gas dynamic case which corresponds to $m_1 = 0$. Thus, the result: for very weak or very strong shocks [in an asymptotic sense], the results are independent of the magnetic field and agree with the usual gas dynamic results.

In magnetohydrodynamics, strong shocks occur for σ close to 4 or for

a very strong applied field for any $\sigma > 1$, i.e., m_1 is then large.

For $m_1 = 0$, K_2 is a monotonically decreasing function of the shock strength and agrees exactly with the corresponding parameter in the afore-mentioned publications.

For any $m_1 \neq 0$, the monotoncity is lost but each curve is concave upward with curves for greater m_1 lying below those for lesser m_1, and all curves pass through the points $(\sigma, K_2) = (1, 0.5)$ and $(4, 0.45085)$.

Further, for fixed incident shock strength, K_2 is a monotonically decreasing function of m_1, i.e., in a diverging (converging) channel, the pressure decrement (increment) is decreased (decreased) by increasing the applied field.

If the area variations are confined to a transition section of finite extent joining two portions of constant area, the total effect of the passage through the transition section is a change in shock strength with the shock eventually again becoming uniform.

Qualitatively, the motion of the shock is independent of whether the main flow behind the shock is $< \omega$ or $> \omega$.

Since the analysis, which is limited to a monatomic gas, includes the usual gas-dynamic results as a special case, there is a check on the theory presented herein.

2. General Theory

The quasi-one-dimensional non-steady flow of an ideal, inviscid, perfectly conducting monatomic compressible fluid, subjected to a transverse magnetic field, i.e., the induction $\vec{B} = (0, 0, B)$, is governed by the system of equations:

$$c_t + uc_x + cu_x/3 + ucA_x/3A = 0 \tag{1}$$

$$3cc_x + u_t + uu_x + b^2B_x/B - 9c^2\eta_x/10c_v = 0 \tag{2}$$

$$B_t + uB_x + Bu_x = 0 \tag{3}$$

$$\eta_t + u\eta_x = 0 \tag{4}$$

where $u, c, \eta, b^2 = B^2/\mu\rho$, ρ, μ and A are, respectively, the particle velocity, local speed of sound, specific entropy, square of the Alfvèn speed, density, permeability and cross-sectional area. Partial derivatives are denoted by subscripts and all dependent variables are functions of x and t alone save A which is considered time independent. The characteristics of this system are:

$$dx/dt = u, u, u + \omega, \; u - \omega$$

where $\omega = [c^2 + b^2]^{1/2}$ which corresponds to the limiting case of a fast wave.

For an arbitrary isentropic constant area flow, the base flow in the neighborhood of which perturbations will be considered, the basic system of equations (1), (2) and (3) may be written in the following characteristic form [3]:

$$x_\beta - (u + \omega)t_\beta = 0 \tag{5}$$

$$x_\alpha - (u - \omega)t_\alpha = 0 \tag{6}$$

$$x_\xi - ut_\xi = 0 \tag{7}$$

$$(\omega^2 - c^2)B_\beta/B + \omega u_\beta + 3cc_\beta = 0 \tag{8}$$

$$(\omega^2 - c^2)B_\alpha/B - \omega u_\alpha + 3cc_\alpha = 0 \tag{9}$$

$$B_\xi/B - 3c_\xi/c = 0 \tag{10}$$

with characteristic parameters (α, β, ξ).

From equation (10), it is clear that the quantity B/c^3 is constant along each particle path, which is a well-known consequence of the assumption of infinite electrical conductivity, i.e., the magnetic field is "frozen" into the fluid. For a constant state or a simple wave flow, B/c^3 is constant throughout the flow, and equations (8) and (9) may be written as:

$$u_\beta + 3\omega c_\beta/c = 0 \tag{11}$$

$$-u_\alpha + 3\omega c_\alpha/c = 0 \tag{12}$$

Since $B = r_1c^3$, $\rho = r_2c^3$ with constants r_1 and r_2,

$$b^2 = r_1^2c^3/\mu r_2 \equiv kc^3 \quad \text{(say)}$$

so that

$$\omega^2 = c^2(1 + kc)$$

and (11) and (12) may be integrated explicitly to yield:

$$u/2 + (1 + kc)^{3/2}/k = u/2 + (\omega/c)^3/k = \alpha \tag{13}$$

$$- u/2 + (1 + kc)^{3/2}/k = - u/2 + (\omega/c)^3/k = \beta \tag{14}$$

where (α, β) may be considered as generalizations of the usual Riemann invariants.

It is convenient to make a transformation of dependent variables so that equations (13) and (14) become linear relations in order to bring the present results into close analogy with ordinary gas-dynamics. This is effected by the substitution $w = (\omega/c)^3$ so that the basic system (1)-(4) becomes:

$$w_t/k + uw_x/k + \omega u_x/2 + u\omega A_x/2A = 0 \tag{15}$$

$$u_t/2 + \omega w_x/k + uu_x/2 - 9\eta_x c^2/20c_v = 0 \tag{16}$$

$$\eta_t + u\eta_x = 0 \tag{17}$$

where it has been noted that

$$B_x/B = 2w_x/k\omega$$

and equation (3) omitted so that the basic system (1)-(4) is effectively reduced to three equations, adding and substracting (15) and (16) gives:

$$u_t/2 + w_t/k + (u+\omega)[w_x/k + u_x/2] + u\omega A_x/2A - 9\eta_x c^2/20c_v = 0 \tag{18}$$

$$- u_t/2 + w_t/k + (u-\omega)[w_x/k - u_x/2] + u\omega A_x/2A + 9\eta_x c^2/20c_v = 0 \tag{19}$$

A formal linearization of the equations (17), (18) and (19) in the neighborhood of a known isentropic solution, denoted by the subscript zero, leads

to the following system of linear equations for the terms of first order, denoted by the subscript one:

$$[w_1/k + u_1/2]_t + (u_1 + \omega_1)[w_0/k + u_0/2]_x + (u_0 + \omega_0)[w_1/k + u_1/2]_x + u_0\omega_0 A_{1x}/2A_0 - 9c_0^2 \eta_{1x}/20c_v = 0 \tag{20}$$

$$[w_1/k - u_1/2]_t + (u_1 - \omega_1)[w_0/k - u_0/2]_x + (u_0 - \omega_0)[w_1/k - u_1/2]_x + u_0\omega_0 A_{1x}/2A_0 + 9c_0^2 \eta_{1x}/20c_v = 0 \tag{21}$$

$$\eta_{1t} + u_0\eta_{1x} = 0 \tag{22}$$

According to (22), η_1 remains constant along the particle paths of the given flow, i.e., along $dx/dt = u_0$. Since $\rho_0(dx - u_0 dt)$ is the exact differential of a function ψ_0 which when equated to a constant, defines the particle paths, the solution of (22) may be written as:

$$\eta_1 = \Omega(\psi_0) \tag{23}$$

with Ω an arbitrary function. It is convenient to define a new function $T_0(x, t)$ by:

$$9c_0^2 \eta_{1x} = 9c_0^2 \rho_0 \Omega'(\psi_0) = 10c_v T_0 \tag{24}$$

Introducing the functions

$$R = u_1/2 + w_1/k \ , \quad S = - u_1/2 + w_1/k$$

and noting that

$$u_0/2 + w_0/k = \alpha \ , \quad - u_0/2 + w_0/k = \beta \ ,$$

the characteristic parameters of the basic flow, equations (20) and (21) may be written as:

$$\begin{aligned} R_t + (u_0 + \omega_0)R_x + [3R - S - (R + S)/3w_0^{2/3}]\alpha_x/2 \\ = T_0/2 - u_0\omega_0 A_{1x}/2A_0 \end{aligned} \tag{25}$$

$$\begin{aligned} S_t + (u_0 - \omega_0)S_x + [R - 3S + (R + S)/3w_0^{2/3}]\beta_x/2 \\ = - T_0/2 - u_0\omega_0 A_{1x}/2A_0 \end{aligned} \tag{26}$$

Equations (25) and (26) have proved useful in discussing perturbations of simple waves. For the present problem, the base flow is a constant state so that α and β are constants, and the general solution in terms of three arbitrary functions of one argument is:

$$T_0 = 2E'[x - u_0 t] \tag{27}$$

$$R = F[x - (u_0 + \omega_0)t] + E[x - u_0 t]/\omega_0 - u_0\omega_0 A_1/2A_0(u_0 + \omega_0) \tag{28}$$

$$S = G[x - (u_0 - \omega_0)t] + E[x - u_0 t]/\omega_0 - u_0\omega_0 A_1/2A_0(u_0 - \omega_0) \tag{29}$$

Thus, there are four distinct contributions to the perturbation, viz., a disturbance due directly to the area variations, one due to the entropy variations, which travels along the particle paths and is measured by E,

a perturbation propagating with velocity ω_0, the true speed of sound, with respect to the fluid along the family of characteristics $x - (u_0 + \omega_0)t = \text{constant}$ and measured by F and a perturbation propagating with velocity ω_0 with respect to the fluid along the family of characteristics $x - (u_0 - \omega_0)t = \text{constant}$ and measured by G.

3. Solution in the vicinity of the incident shock

Jump conditions across normal magnetohydrodynamic shocks have been considered by several authors, e.g., Friedrichs [4]. Let U be the shock velocity, $v = U - u$ and the subscripts one and two designate flow quantities in the regions in front of and behind the shock. Then the analogs of the Rankine-Hugoniot relations are:

$$\rho_1 v_1 = \rho_2 v_2$$

$$B_1 v_1 = B_2 v_2$$

$$\rho_1 v_1^2 + P_1 + B_1^2/2\mu = \rho_2 v_2^2 + P_2 + B_2^2/2\mu$$

$$\gamma P_1/(\gamma - 1)\rho_1 + v_1^2/2 + B_1^2/\rho_1\mu = \gamma P_2/(\gamma - 1)\rho_2 + v_2^2/2 + B_2^2/\rho_2\mu$$

For gas-dynamics shocks, knowledge of the flow in front of and one parameter behind the shock suffices to give a complete solution. In magnetohydrodynamics, all quantities behind the shock may be expressed in terms of those in front and two parameters, viz., the shock strength and one which gives a measure of the applied field.

Let $m = b/c$, $n = u/c$, $\sigma = \rho_2/\rho_1$, $\tau = P_2/P_1$, $M = v/c$ and

$q^2 = \omega^2/c^2 = 1 + m^2$. Then the following relations hold, where n_1 is assumed zero:

$$v_1/v_2 = \rho_2/\rho_1 = B_2/B_1 = b_2^2/b_1^2 = \sigma \tag{30}$$

$$\tau = [4\sigma - 1 - 5m_1^2(1-\sigma)^3/6]/(4-\sigma) \tag{31}$$

$$M_1^2 = [6\sigma + m_1^2\sigma(5+\sigma)]/2(4-\sigma) \tag{32}$$

$$\begin{aligned} n_2 &= (1-\sigma^{-1})M_1(\sigma/\tau)^{1/2} \\ &= (\sigma - 1)\left\{[3 + m_1^2(5+\sigma)/2]/[4\sigma - 1 - 5m_1^2(1-\sigma)^3/6]\right\}^{1/2} \end{aligned} \tag{33}$$

$$c_2^2/c_1^2 = \tau/\sigma \tag{34}$$

$$m_2^2/m_1^2 = \sigma^2/\tau \tag{35}$$

$$M_2^2 = M_1^2/\sigma\tau \tag{36}$$

which express the flow parameters behind the shock in terms of σ and m_1.

The effect of an area variation on the motion of an initially uniform shock propagating with constant speed into a fluid at rest will now be determined by the use of the general solution for the non-isentropic perturbation of a constant state, equations (27)-(29). The method of generation of the shock is left open save that whatever that may be, the mechanism is sufficiently far removed so that no reflections come back to interfere with the basic interaction considered herein.

From the formulation of the problem, there is no mechanism downstream of the shock which could give rise to the term $F[x - (u_2 - \omega_2)t]$ in (28). The

pressure perturbation behind the shock will be determined by setting $F = 0$. This gives:

$$\bar{u}_2/2 + \bar{w}_2/k - 9c_2^{\,2}\bar{\eta}_2/20\omega_2 c_v = - u_2\omega_2\bar{A}_2/2A_2(u_2 + \omega_2) \tag{37}$$

where perturbations of a quantity are denoted by a bar. Since

$$\bar{w}_2/k = 3\omega_2\bar{c}_2/2c_2$$

and

$$- 9\bar{\eta}_2/20c_v = 3\bar{P}_2/10P_2 - 3\bar{c}_2/2c_2 \ ,$$

the latter from the equation of state, equation (37) may be written as:

$$\bar{u}_2/2 + 3\bar{c}_2 m_2^{\,2}/2q_2 + 3c_2\bar{P}_2/10q_2P_2 = - u_2q_2\bar{A}_2/2(n_2 + q_2)A_2$$

or

$$\bar{u}_2/c_2 + 3[5m_2^{\,2} + 2]\bar{P}_2/10q_2P_2 - 3m_2^{\,2}\bar{\rho}_2/2q_2\rho_2 = - n_2q_2\bar{A}_2/(n_2 + q_2)A_2 \tag{38}$$

From the jump conditions, equations (30)-(36),

$$\frac{\bar{\tau}}{\tau} = \frac{[90\sigma + 5m_1^{\,2}\sigma(11 - 24\sigma + 15\sigma^2 - 2\sigma^3)]}{(4 - \sigma)[6(4\sigma - 1) - 5m_1^{\,2}(1 - \sigma)^3]}\ \frac{\bar{\sigma}}{\sigma}$$

$$\frac{\bar{u}_2}{c_2} = \frac{[24 + 12\sigma + m_1^{\,2}(20 + 10\sigma + 7\sigma^2 - \sigma^3)]}{2(4 - \sigma)(\sigma - 1)[6 + m_1^{\,2}(5 + \sigma)]}\ \frac{n_2\bar{\sigma}}{\sigma}$$

so that (38) gives:

$$\bar{P}_2/P_2 = - K_1[\sigma, m_1]\bar{A}_2/A_2 \tag{39}$$

where:

$$K_1 = \frac{n_2 q_2}{n_2+q_2}\left[\frac{90\sigma+5m_1^2\sigma(11-24\sigma+15\sigma^2-2\sigma^3)}{6(4\sigma-1)-5m_1^2(1-\sigma)^3}\right]\left\{n_2\left[\frac{24+12\sigma+m_1^2(20+10\sigma+7\sigma^2-\sigma^3)}{2(\sigma-1)[6+m_1^2(5+\sigma)]}\right]+\right.$$

$$\left.\frac{3(5m_2^2+2)}{10q_2}\left[\frac{90\sigma+5m_1^2\sigma(11-24\sigma+15\sigma^2-2\sigma^3)}{6(4\sigma-1)-5m_1^2(1-\sigma)^3}\right] - \frac{3m_2^2(4-\sigma)}{2q_2}\right\}^{-1}$$

or

$$\bar{P}_2 = -K_2(\sigma, m_1)(P_2 - P_1)\bar{A}_2/A_2 \tag{40}$$

where

$$K_2 = \left[\frac{5m_1^2(1-\sigma)^3 - 6(4\sigma-1)}{30(1-\sigma) + 5m_1^2(1-\sigma)^3}\right] K_1$$

Strong shocks can occur in magnetohydrodynamics in two ways, viz., for σ close to 4 or for a very strong applied field for any $\sigma > 1$, i.e., m_1 is then large.

For all m_1, $\lim_{\sigma\to 1+} K_2 = 0.5$ and $\lim_{\sigma\to 4-} K_2 = 0.45085$, but these are precisely the limits for the ordinary gas dynamic case which corresponds to $m_1 = 0$. Thus, for very strong or very weak shocks [in an asymptotic sense], the results are independent of the magnetic field and agree with the usual gas dynamic results.

For $m_1 = 0$, K_2 decreases monotonically with σ and agrees exactly with the corresponding parameter in the gas dynamic case [2].

For any $m_1 \neq 0$, there no longer is a monotonic variation with σ, but

the curves are all concave upward with those for greater m_1 lying beneath those for lesser m_1. All the curves issue from the point $(\sigma, K_2) = (1, 0.5)$ and terminate at the point $(4, 0.45085)$.

For fixed incident shock strength σ, K_2 decreases monotonically with increasing m_1, so that in a diverging (converging) channel, the pressure decrement (increment) is decreased (decreased) by increasing the applied field.

Qualitatively, the motion of the shock is independent of whether the main flow behind the shock is $< \omega$ or $> \omega$.

Graphs of K_2 are given in Figure 1.

4. The Reflected Disturbance

To complete the solution, the arbitrary function G of equation (29) must be determined. This is done most readily by noting that the system (27)-(29) may be written as:

$$\bar{u}_2/2 + 3\bar{c}_2 m_2^2/2q_2 + 3c_2\bar{P}_2/10q_2P_2 = - u_2 q_2 \dot{\bar{A}}_2/2A_2(n_2+q_2) - \bar{u}_2/2 + 3\bar{c}_2 m_2^2/2q_2$$

$$+ 3c_2\bar{P}_2/10q_2P_2 = - u_2 q_2 \bar{A}_2/2A_2(n_2 - q_2) + G[x - (u_2 - \omega_2)t]$$

Thus, on addition:

$$\left\{ \frac{15m_2^2+6}{10q_2} - \frac{3m_2^2(4-\sigma)}{2q_2\sigma} \frac{[6(4\sigma-1)-5m_1^2(1-\sigma)^3]}{[90+5m_1^2(11-24\sigma+15\sigma^2-2\sigma^3)]} \right\} \frac{\bar{P}_2}{P_2}$$

$$+ \frac{n_2^2 q_2}{n_2^2-q_2^2} \frac{\bar{A}_2}{A_2} = G[x - (u_2 - \omega_2)t]/c_2$$

Evaluating this on the shock, $x = Ut$, and replacing $\bar{P}_2/P_2$ by its value from (39) gives:

$$G[\lambda]/c_2 = K_3\bar{A}_2[\delta\lambda]/A_2$$

where

$$\lambda = x - (u_2 - \omega_2)t$$

$$\delta = (M_2 + n_2)/(M_2 + q_2)$$

$$K_3 = \frac{n_2^2 q_2}{n_2^2 - q_2^2} - \frac{3K_1}{2q_2}\left[\frac{5m_2^2+2}{5} + \frac{m_2^2(4-\sigma)}{\sigma}\left\{\frac{5m_1^2(1-\sigma)^3 - 6(4\sigma-1)}{90+5m_1^2(11-24\sigma+15\sigma^2-2\sigma^3)}\right\}\right]$$

Thus:

$$\bar{P}_2/(P_2-P_1) = -\xi_1\bar{A}_2[x]/A_2 - \xi_2\bar{A}_2[\delta\lambda]/A_2 \tag{41}$$

where

$$\xi_1 = \frac{\dfrac{n_2^2 q_2}{n_2^2-q_2^2}\left[\dfrac{5m_1^2(1-\sigma)^3-6(4\sigma-1)}{30(1-\sigma)+5m_1^2(1-\sigma)^3}\right]}{\dfrac{3}{2q_2}\left[\dfrac{5m_2^2+2}{5} + \dfrac{m_2^2(4-\sigma)}{\sigma}\left\{\dfrac{5m_1^2(1-\sigma)^3 - 6(4\sigma-1)}{90+5m_1^2(11-24\sigma+15\sigma^2-2\sigma^3)}\right\}\right]}$$

$$\xi_2 = -\xi_1 + K_2$$

The parameter δ is a monotonic increasing function of the shock strength and varies between the limits

$$0.5 \leq \delta \leq 1.2361$$

for all m_1. Graphs are presented in Figure 2.

It might be noted that Chester's graph of the corresponding function is incorrect. He gives it as a monotonic decreasing function and the value for a very weak shock ($\sigma = 1$) is the reciprocal of the true value.

When the flow behind the shock is $< \omega$, a disturbance will be reflected to the left downstream of the shock, and, from (41), the pressure perturbation will ultimately be given by:

$$\bar{P}_2 = - \xi_2 \bar{A}_2 [\delta \lambda] / A_2$$

since $\bar{A}_2 = 0$ for $x < 0$.

For flow $> \omega$ behind the incident shock, the reflected disturbance will travel to the right and the pressure perturbation is given by (41).

Graphs of the parameter ξ_2 are presented in Figure 3. The parameter is singular for $n_2 = q_2$. This difficulty may be eliminated by a more careful consideration of the basic perturbation equations by retention of appropriate nonlinear terms. However, since the main purpose of this paper is to consider the pressure perturbation in the immediate neighborhood of the incident shock, such are not employed.

5. Concluding Remarks

In conclusion, let it be noted that a theory has been developed rather parallel to that of Chisnell [2] for the magnetohydrodynamic case, and this will be reported when the numerical computations have been completed.

For the present, suffice it to say that this theory gives that the strength of converging cylindrical and spherical magnetohydrodynamic shocks near the point of collapse is proportional to $D^{-K_{2\infty}}$ and $D^{-2K_{2\infty}}$, respectively,

independent of the applied field, where $K_{2\infty} = 0.45085$ and D is the distance from the point of collapse.

The author wishes to thank Mr. J. Al-Abdulla for carrying out the computations employed herein.

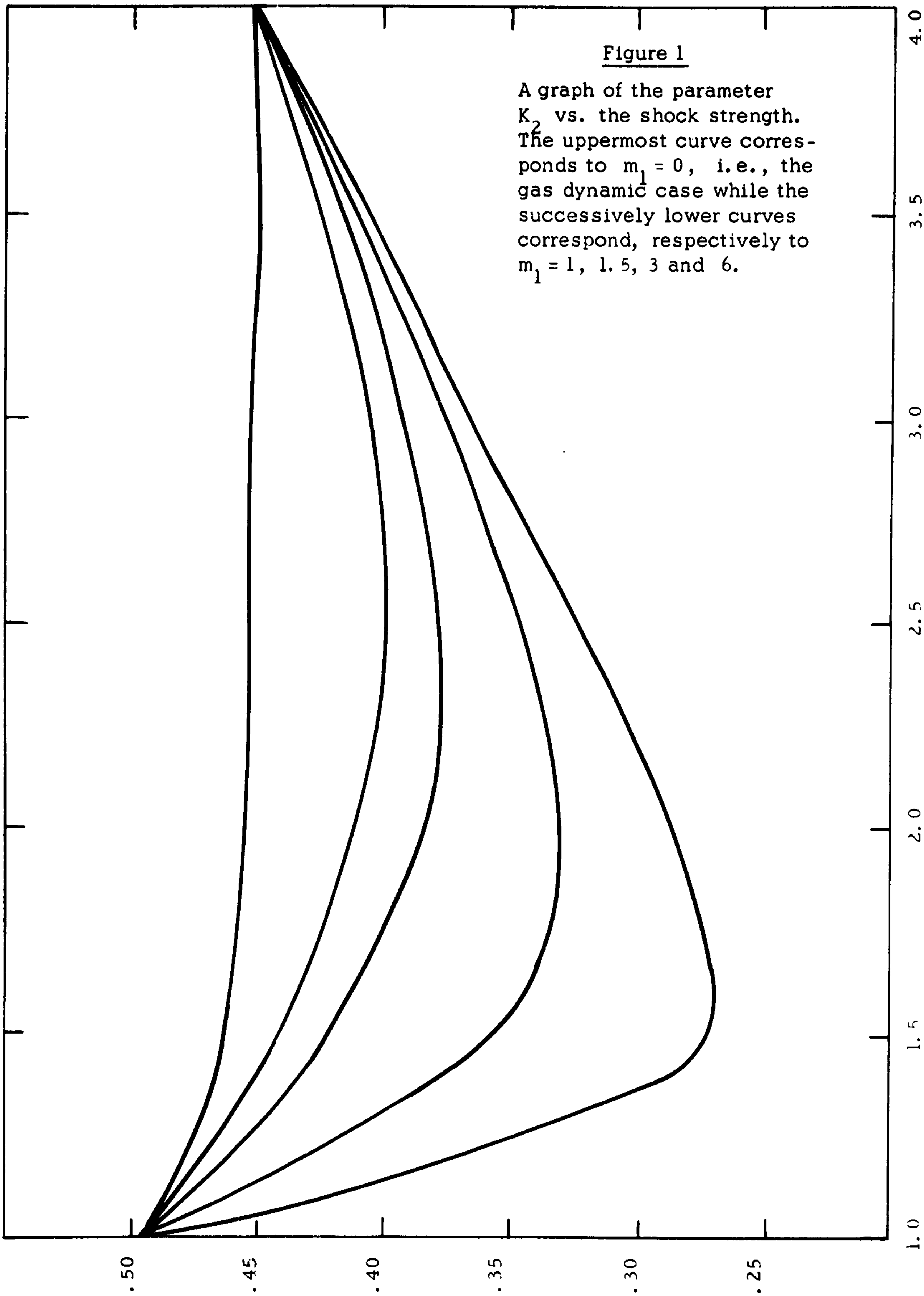

Figure 1

A graph of the parameter K_2 vs. the shock strength. The uppermost curve corresponds to $m_1 = 0$, i.e., the gas dynamic case while the successively lower curves correspond, respectively to $m_1 = 1$, 1.5, 3 and 6.

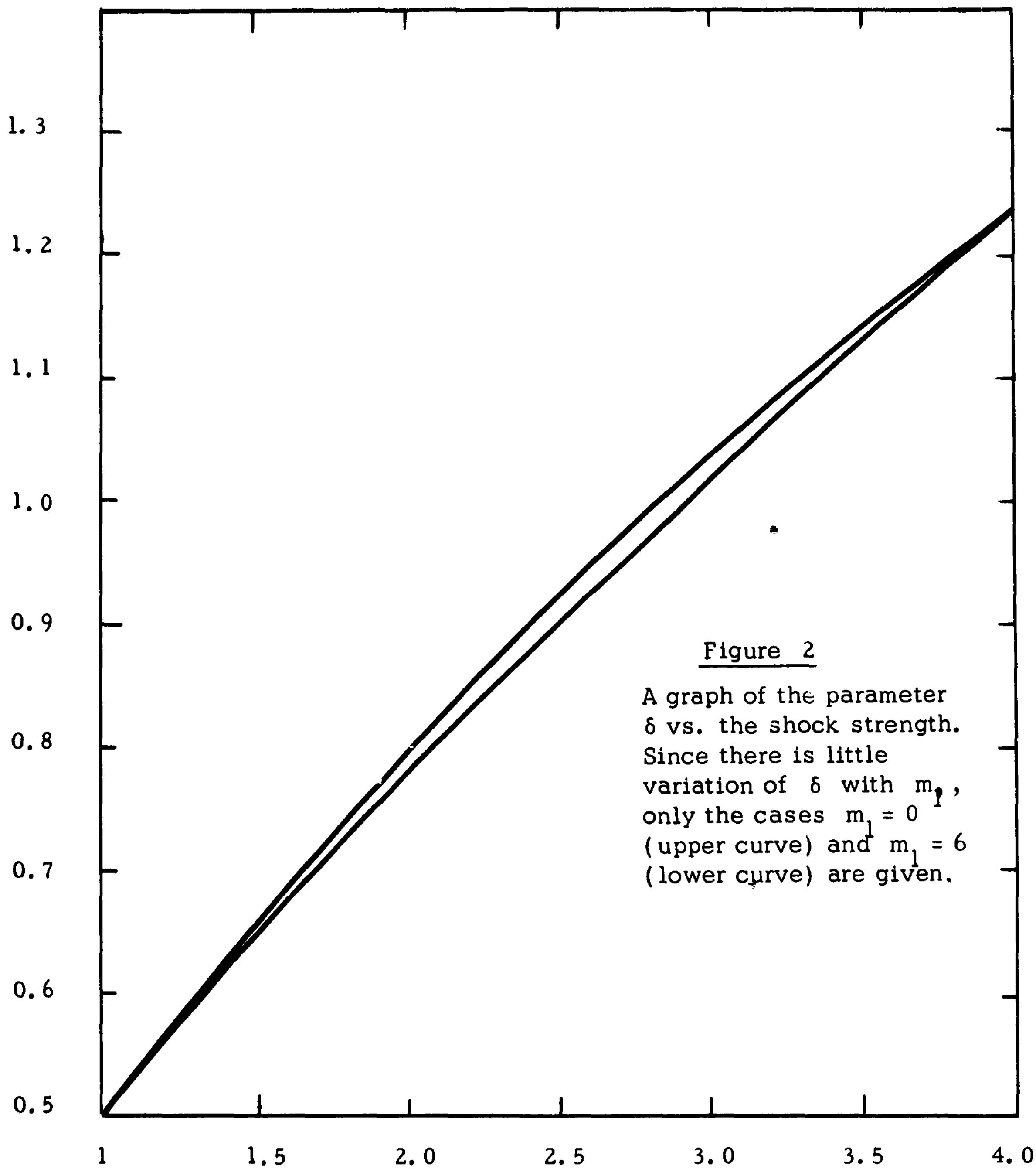

Figure 2

A graph of the parameter δ vs. the shock strength. Since there is little variation of δ with m_1, only the cases $m_1 = 0$ (upper curve) and $m_1 = 6$ (lower curve) are given.

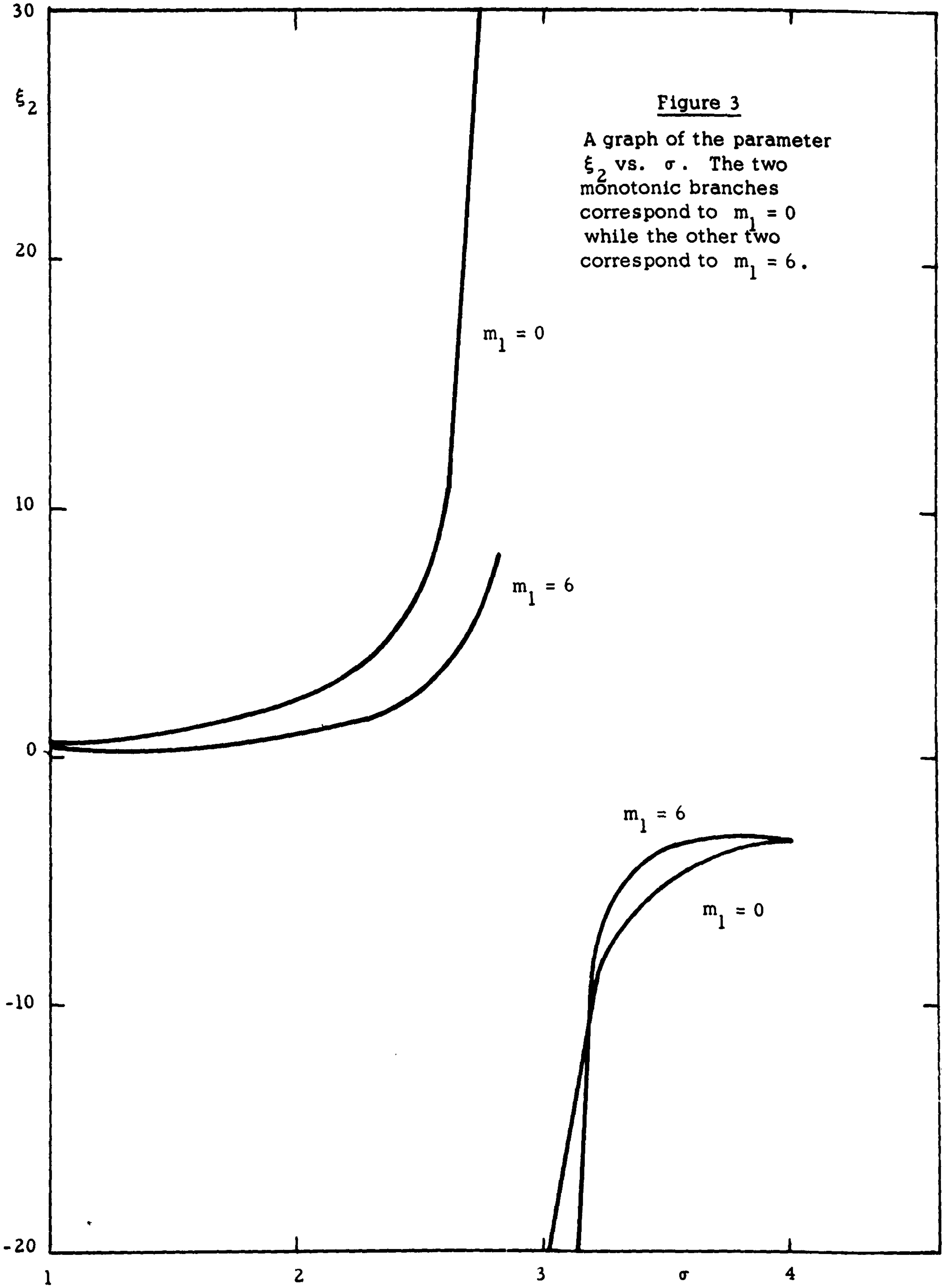

Figure 3

A graph of the parameter ξ_2 vs. σ. The two monotonic branches correspond to $m_1 = 0$ while the other two correspond to $m_1 = 6$.

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